Mathematics Education for the Third Millennium: Visions of a future for handheld classroom technology

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Research into the impact and effectiveness of handheld technologies in the teaching and learning of mathematics shows signs of having matured over the past few years. This "coming of age" will prove useful in the years ahead. Significant developments in the nature and purposes of the technology demand new questions and new approaches from those that would chart a course for others to follow in their effective uptake. This paper addresses some of these new questions and approaches within the context of the tools that drive them. It proposes a research agenda which is mindful of previous "blindspots" as well as some new imperatives imposed by radical developments in the technology itself.

Introduction

Few working in mathematics education today would be unaware of the growth in significance in recent years of handheld technologies for teaching and learning. The increased educational profile of technology which includes calculators of all types, but particularly graphing calculators and their peripherals, is neatly acknowledged by our own latest MERGA review of recent research. For the first time "Calculators and Computer Algebra Systems" is designated its own chapter (Forster, Flynn, Frid, & Sparrow, 2004), distinct from a consideration of "Computers, Multimedia, and the Internet in Mathematics Education" (Goos & Cretchley, 2004). It would appear that these "little toys for mathematics learning" may have "come of age" and be worthy of quite specific consideration, particularly with regard to research.

This paper examines key questions associated with research into the use of handheld technologies for the teaching and learning of mathematics. Such questions relate to the nature and directions of the research itself, within the context of the current and future capabilities of the technology. The principle focus will be upon graphing calculators and their related technologies, which will be referred to as handhelds, rather than calculators to distinguish a broader range of functionality now available. It is not proposed here to review the current research literature in this domain in any detail, since this has been ably and extensively done by others, (in particular, Burrill et al., 2003). Rather, the intention is to cast a critical eye over the research domain itself, with an eye for two specifics: the possibility of what might be termed "research blindspots" and imperatives dictated by advances in the technology itself. In this way, we may realistically look towards the year 2010 and be afforded at least a glimpse of what such a future may hold for teachers and learners of mathematics.

The competence of the author to offer such a viewpoint may be judged against three particular elements of experience brought to the task. Eight years ago, I was fortunate enough to be involved in an extensive critical review of graphic calculator research to date, which proved timely and useful in drawing attention to what might now be considered a few "growing pains" of a fledgling field of enquiry (Penglase & Arnold, 1996). For five of the subsequent years since then, I returned to classroom teaching, making daily use of this technology with classes at all levels across years 7 to 12. Finally, most of the past twelve months has been spent working with teachers across the country and overseas within a professional development context, supporting the uptake of handheld technology for mathematics teaching and learning. These three perspectives, of researcher, classroom teacher and teacher educator, support the contention that the vision offered here may be considered current and feasible.

The State of Play of Current Research

Any review of research and publication in this domain quickly reveals very positive features. Indeed, there appears almost an urgency in the flood of literature available to support practitioners and policy makers in their decision-making regarding planning and use related to handheld tools for mathematics classrooms. A search of the *Digital Dissertations Abstract* on-line database of theses and dissertations since 1996, under the criteria “graphic calculator” OR “graphics calculator” OR “graphing calculator” produced 138 citations. A similar search of the *Proquest* on-line database of refereed journal articles and reports returned over 440 citations. Such interest may be better understood in the context of a national survey of United States high schools in 2000, which revealed that over 80% of high school teachers of mathematics made use of handheld graphing technology in their classrooms (Weiss, Banilower, & Smith, 2001). While we are lacking in such informative data for Australia, it is certainly true that this year, for the first time, every state and territory has permitted the use of handheld graphing technology in senior years and high-stake external examinations (where applicable) for students of mathematics. It would be very useful to know what percentage of classrooms across Australia are making use of these tools, and in what ways: there is clear scope for a major research project which would paint such a picture for us. Even in the absence of such data, however, it is clear that a quiet revolution has occurred in recent years, from a perception of handheld mathematical tools as being “optional extras” to a position where they are considered integral components in mathematics learning. Both here and overseas, the face of mathematics classrooms has been forever changed through access to these tools.

But how are they being used? What has research to say with regard to effective use and, indeed, the very nature of such use? Certainly, the mistakes of earlier studies are less common now: questions related to measuring the “effectiveness” of graphic calculators, as if such devices exist in isolation from their curricular and pedagogical contexts, are no longer prevalent. Quality research which utilises both qualitative and quantitative components, spanning substantial samples over significant time-frames has become less of a rarity. Australian work appears at the forefront of research worldwide, and is regularly cited in reviews.

Texas Instruments in 2002 commissioned an extensive review of research worldwide on handheld graphing technology, in association with Michigan State University, under the directorship of former NCTM President, Gail Burrill (Burrill et al., 2003). Initially identifying over 180 peer-reviewed published research reports, the project team narrowed the field down to 43 studies which were deemed to fulfil the strict criteria for inclusion. These criteria related to publication, perceived relevance, inclusion of evidence, rigour and scientific design. It should be noted that when the team further applied the strict criteria for “scientifically based research” as described in the “No Child Left Behind” Act, the final pool consisted of only six studies.

It is of some relevance here to list the five focus questions identified by the review, based upon the 43 studies. The team found that the quality research studies identified clearly grouped around the following key questions:

1. How do teachers use handheld graphing technology and how is this use related to their knowledge and beliefs about technology, mathematics, and teaching mathematics?
2. **12 studies:** (Doerr & Zangor, 1999, 2000; Farrell, 1996; Fleener, 1995a, 1995b; **Goos, Galbraith, Renshaw, & Geiger, 2000;** Harskamp, Suhre, & van Streun, 1998, 2000; Lloyd & Wilson, 1998; Rochowicz Jr., 1996; Slavit, 1996; Tharp, Fitzsimmons, & Ayers, 1997)
3. With what kind of mathematical tasks do students choose to use handheld graphing technology?
4. **14 studies:** (Berger, 1998; **Boers & Jones, 1994**; Dahland & Lingefjard, 1996; Doerr & Zangor, 2000; Drijvers & Doorman, 1996; **Forster & Mueller, 2001**; Guin & Trouche, 1998; Hennessy, Fung, & Scanlon, 2001; **Hong, Thomas, & Kiernan, 2000**; Keller, 1998; Lauten, 1994; **Mitchelmore & Cavanagh, 2000**; Ruthven, 1990; Schwarz & Hershkowitz, 1999)
5. What mathematical knowledge and skills are learned by students who use handheld graphing technology?
6. **23 studies:** (Adams, 1997; Connors & Snook, 2001; **Forster, 2000; Forster & Taylor, 2000**; **Graham & Thomas, 1998, 2000**; Harskamp et al., 1998, 2000; Hollar, 1996; **Hong et al., 2000;** Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000; Keller & Hirsch, 1998; Keller, Russell, & Thompson, 1999; Kendal & Stacey, 1999; Lauten, 1994; Merriweather & Tharp, 1999; Quesada & Maxwell, 1994; Ruthven, 1990; Schwarz & Hershkowitz, 1999; Slavit, 1998; Thompson & Senk, 2001; van Streun, Harskamp, & Suhre, 2000; Zbiek, 1998)
7. What is gained mathematically by students using handheld technology that cannot be observed in a non-technology environment?
8. **21 studies:** (Adams, 1997; Drijvers, 2000; Drijvers & Doorman, 1996; Drijvers & van Herwaarden, 2000; Farrell, 1996; **Forster, 2000**; Graham, 1998; Guin & Trouche, 1998; Harskamp et al., 1998, 2000; Hennessy et al., 2001; Hollar & Norwood, 1999; **Hong et al., 2000**; Huntley et al., 2000; **Mitchelmore & Cavanagh, 2000**; Porzio, 1999; Quesada & Maxwell, 1994; Ruthven, 1990; Schwarz & Hershkowitz, 1999; Slavit, 1996, 1998; Thompson & Senk, 2001)
9. Does handheld graphing technology have similar effects on the performance of students from different gender, socio-economic status and achievement groups? **6 studies:** (Forster & Mueller, 2001; Harskamp et al., 2000; Hollar, 1996; Hollar & Norwood, 1999; Hong et al., 2000; Ruthven, 1990; Shoaf-Grubbs, 1992)

We may (briefly) allow ourselves a little national pride in the relative frequency of Australian and New Zealand researchers (as shown in bold-face) in this “honour-roll” of good research as judged against an international field.

It is appropriate at this point to engage the issues at a slightly higher level of abstraction, and consider our research priorities. Consider, for example, the numbers of studies cited in each of these five key categories: why so many for questions 3 and 4, and so relatively few for question 5? While acknowledging that the particular selection process for these references may well have influenced the numbers, it draws us to consider the next critical issue regarding our on-going research agenda in this domain: what do we value in this field of enquiry, and how is this reflected in our research agenda? Are we asking the questions which truly need to be asked, or those which are perhaps more accessible or even more likely to bear fruit?

In an extensive search of available databases and relevant research sources for this paper, a collection of some 580 citations was accumulated which referred to handheld graphing tools ("graph* calculator"). Of these, 219 made reference to "algebra"; 137 involved "functions"; 136 referred to "calculus". In both the extensive Burrill review and our own MERGA review of research related to handheld graphing technology, it was observed without comment that the vast majority of studies in this domain were in the fields of algebra and calculus.

What then of statistics? Once again, in Australia, every state and territory (except one) offers a substantial statistical component of senior mathematical study to at least their high-ability students. The capabilities of all modern handheld graphing tools offer advanced but accessible statistical facilities which have transformed the teaching of this discipline at all levels. Access to appropriate technology has changed the focus of this domain from one of computation to one of inference. The majority of senior students now studying statistics use a graphing calculator to assist in their work.

So what of the research into student use of this technology for the learning of statistics? What of the concept development so central to this domain? There are more concepts in the learning of mathematics than function, variable and derivative, but this is not reflected in the research. Of the 138 graphing calculator dissertations found in *Digital Dissertations*, only *four* were associated with the teaching and learning of statistics. Of these, two were comparative studies between graphing calculators and computer software: spreadsheet software (Creed, 2000) and an interactive statistics package, Fathom (Emerson, 2000). The other two studies related to reasoning about probability distributions during instruction (Zimmermann, 2002) and data analysis to explore the sampling distribution of the sample mean (Carson, 1995). Of the 440 journal articles and reports drawn from the *ProQuest* database, only 50 referred to statistics in the context of handheld graphing tools.

The significance of the role of technology in the teaching and learning of statistics is not in dispute. As far back as 1996, the International Association for Statistical Education held a Round Table Conference in Grenada, Spain, on the theme, *Research on the Role of Technology in Teaching and Learning Statistics*. Keynote papers included one by Gail Burrill on "Graphing calculators and their potential for teaching and learning statistics" (Burrill, 1996). Note that this is the same Gail Burrill who, as director of the *Texas Instruments* review of research did not find any research studies of statistics involving graphing calculators worthy of inclusion in the review.

It is difficult to understand this omission other than as a "research blindspot". Our perception of these devices is primarily as tools for algebra and calculus learning, and this limits our view of appropriate research questions. The result of this blindspot is that, with regard to teaching and learning statistical concepts and skills, we are working in the dark, without any form of research base.

Another potential research blindspot is worth considering. When travelling around the nation last year, talking to teachers, academics and policy makers about the uptake and use of handheld technology in their states and territories, the response regarding New South Wales was always the same: a very critical response to the availability of graphing calculators only for the less capable senior students of mathematics, and denied to the most capable.

And yet who is likely to benefit more from access to these powerful tools? There has been much talk over the years of technology potentially offering a more level playing field, and this question becomes even more relevant in the context of computer algebra systems. But who remain the principal users of the technology, across Australia and overseas? Once again, our perception of the nature and purpose of these tools may be blinding us to important and relevant research questions. We see them as high-powered tools for the most capable senior students of mathematics. The research is equivocal regarding the effects of access to handheld graphing technology among students of different achievement levels (Hong et al., 2000). There is a real need for further research which examines questions related to the "level playing field" hypothesis. To this might well be added the Vygotskian question: *to what extent do these tools extend the cognitive reach of their users?*

An almost incidental comment among recommendations for further research from the Burrill study noted that there was little evidence of research on the use of graphing calculators among years 7 to 9; the focus was almost entirely on years 10 to 12. At the same time, the national survey of nearly 6 000 mathematics and science teachers revealed that, in addition to the nearly 80% of years 9-12 teachers who had used graphing calculators in their most recent lessons, 39% of teachers of years 5-8 had also done so (Weiss et al., 2001). While not high, this figure is not insignificant, and points to a substantial uptake of the technology at very early levels. Once again, there is value in questioning our assumptions regarding the "proper place" for these tools and, in so doing, new and significant research questions may arise. It is on this note that the focus of this paper shifts from the present and past to the future, in terms of the newer capabilities of these tools and their implications and imperatives for future research.

New Tools for a New Millennium

In deciding on the focus for this paper, it seemed worthwhile to examine the changes since the author last reviewed the state of research in this field, in 1996 (Penglase & Arnold, 1996). The preceding section has raised questions related to a research agenda based upon a technology which has been more or less stable for a decade, and relatively little different to the tools which first appeared nearly twenty years ago. In fact, the vast majority of the research cited above still grapples with the original question: *what should a teacher do to make best use of a device which graphs functions?* Few of the many powerful financial or statistical capabilities have been examined; even the more sophisticated algebraic features (equation solving, numerical derivatives, areas under curves, matrix and vector capabilities) have rarely been the subject of explicit study. The programming capabilities and the wonderful list management features as tools for concept development and skill consolidation are certainly worthy of some attention. Our research agenda appears so fixated on algebra, function and calculus questions that we are unable to move on until these have been resolved!

When considering the changes in the actual technology over the past five years in particular there is now an urgent need to reassess our thinking and priorities regarding these handheld tools. In fact, the changes have been so dramatic and substantial that they cross over boundaries into areas previously charted by other explorers, as well as into fields where no-one has yet trodden. In this second section, four particular aspects of new handheld technology will be briefly discussed:

1. Possibilities for CAS (computer algebra systems)
2. Implications for data loggers in the mathematics classroom
3. New software capabilities for these handheld tools
4. Implications of networking capabilities

The focus will be on *Texas Instruments* technology, since their research and development has led the field in exploring these new directions. There appears little doubt, however, that others will follow, and that the innovations described here will likely feature in mathematics classrooms of the not-too-distant-future.

Symbolic Algebra Capabilities

Computer algebra capabilities have been available for desktop computers and even graphic calculators since the late 1980s; the version available now for *Texas Instruments* CAS models has been almost unchanged since it was released for the original TI-92 in the mid-90s. It has taken almost a decade for this powerful mathematical feature to be widely and systematically incorporated in both mathematics classrooms and research. Even today, there remain widespread reservations regarding, not only the best ways to incorporate such tools into teaching and learning, but fundamental questions regarding their appropriateness for inclusion in secondary schools. This last includes a paper charmingly titled *"Education or CAStration?"* (Gardiner, 2001).

Significant progress has been made in this regard through the partnership between the University of Melbourne, the Victorian Curriculum and Assessment Authority and three calculator industry partners over the past three years, as a system-wide investigation has been pioneered (Ball, 2002; Ball & Stacey, 2003; Ball, Stacey, & Pierce, 2001; Leigh-Lancaster, 2000; Pierce & Stacey, 2001; Stacey, Kendal, & Pierce, 2002). Long-term questions have been confronted in this extensive and ground-breaking series of studies, particularly those regarding appropriate roles for CAS: as assistant primarily for *doing* mathematics, or capable of also assisting in the teaching and learning process? Just as significant have been the new questions revealed, such as the extent and nature of solution formats in a CAS-assisted environment.

We are in a position to learn much from the lessons of the past. We may recognise that the questions which practitioners have been asking regarding CAS in secondary classrooms (*What will be left to teach if students have access to tools which factorise, solve, and do calculus? What about their manipulative skills? What will we ask them to do in examinations?*) were the same questions asked a few years ago regarding graphing calculators. In fact, they were precisely the same questions asked twenty years ago and more regarding student access to traditional calculators. We have learned, perhaps, that it is possible to put mathematical power in the hands of students and derive great educational benefit from considered use of it.

We may even be in a position to confront some of the big political questions which arise concerning this issue. Algebra remains very much a sacred cow for mathematics and education systems in general. It has long served a gatekeeper role, deciding who will qualify to study higher levels of mathematics and, consequently, who will master the skills most valuable in our increasingly-technological society.

But what of those students who possess good mathematical insight and understanding, but not perhaps the automated skills of manipulation we have demanded in the past? Can we, as a society, continue to deny access in those areas of study most relevant to the maintenance and advancement of our society to vast numbers of students who are willing, and potentially able (with a little technological support)? It is common today to find students in senior mathematics classes whose number skills would, in days past, have denied them access to any study of mathematics at senior level. Now, with some assistance from technology, they demonstrate that not knowing your tables does not necessarily mean you are incapable of understanding and pursuing further mathematics.

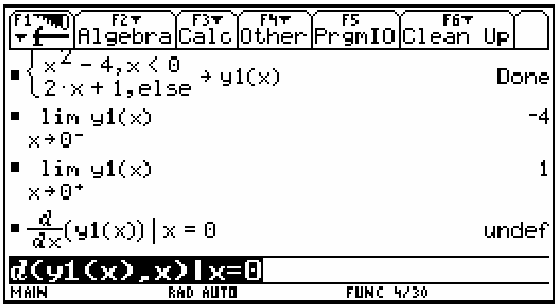


Figure 1: Handheld computer algebra

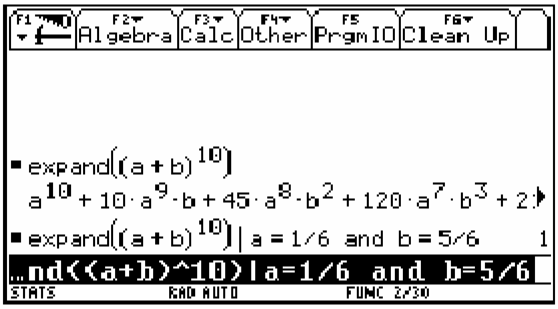


Figure 2: Discrete or continuous?

The writing is on the wall: CAS has a place in the future of mathematics teaching and learning. It remains only for us to make decisions regarding what that place should be and, as always, to design research which may serve to illuminate the path ahead.

Data Logging in the Mathematics Classroom

A second significant area of development related to handheld graphing technology over the past decade has been the increasing presence of data-loggers in mathematics classrooms. Most obvious in those places where teachers of mathematics are also commonly also teachers of science, there is growing evidence that access to real-world data can be effective in the teaching of at least algebra and calculus concepts, and for the modelling of key mathematical functions (Arnold, 1999; Bosche, 1997; Brueningsen & Bower, 1995; Caldwell, 1997; Cates, 2000; Green, 1998; Haruta, Turpin, & McGivney, 1998; Nicol, 1995; White & Norwich, 1997).

Once more, we find that research is lacking in this domain. Only three research dissertations in the past decade have considered the role of data-logging as a powerful mathematical tool for learning (Bosche, 1997; Cates, 2000; Nicol, 1995). Like our consideration of computer algebra, this is hardly a new capability of these tools, merely a neglected one. Unlike the CAS technology, the use of data-loggers potentially extends from the senior years of study down across the high school years and even potentially into the primary school. From the simple but powerful applications of motion detectors at all levels of schooling to the possibilities for introducing functions from multiple perspectives using probes as diverse as microphones and light meters, teachers using these tools in their classrooms have no doubts concerning their appropriateness and potential.

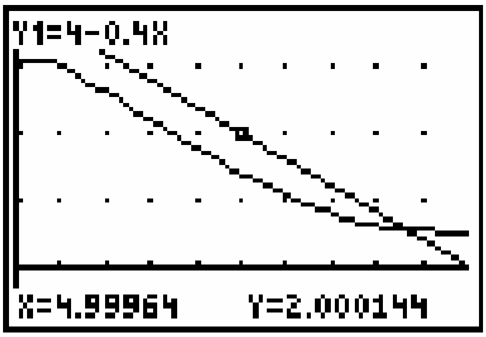


Figure 3: Matching motion to function

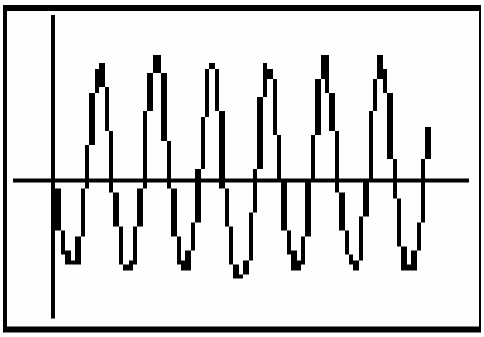
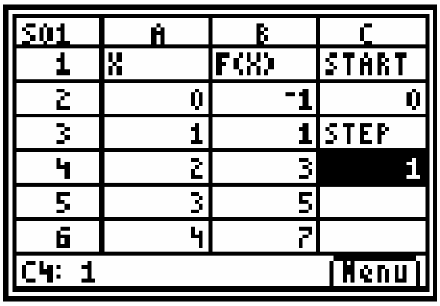


Figure 4: Generating functions

Let is add one more item to our growing list of research imperatives for the coming few years! Perhaps the most significant shift in mathematics curriculum over the past two decades has been that from the continuous to the discrete, dislodging the study of the calculus from its traditional place as the pinnacle and primary goal of school mathematics. As mentioned previously, this coveted position is now shared between the study of functions and that of data, between calculus and statistics. And no clearer nor more effective bridge exists between these two domains than the generation and modelling of functions using real-world data.

Handheld Technology Across the Curriculum

The extension of the capabilities of handheld graphing tools in recent years to enable them to run sophisticated software, as does a computer, sees the convergence of the two technologies, desktop and handheld. An extensive range of over seventy software “Apps” is currently available for *Texas Instruments* calculators, most of these free and appropriate for classes from upper Primary to senior studies. They have the potential to redefine the device, and include, for mathematics teaching and learning, an *Excel*-compatible spreadsheet and a version of *Cabri-Geometry II*, the latter free for the popular *TI-83Plus* calculator. Research on both applications of spreadsheets and interactive geometry have, in the past, involved computer-based studies: from *Digital Dissertations Abstracts* since 1996, 58 citations for spreadsheets and 19 for “dynamic geometry” applied to mathematics education. The ability to run versions of these well-established software tools on handheld devices already present in many mathematics classrooms worldwide significantly extends the capabilities of these already-powerful tools for learning. It also creates enormous new possibilities for student access at home and school to learning opportunities previously only available in a computer room.



S01	A	B	C
1	X	F(X)	START
2	0	⁻1	0
3	1	1	STEP
4	2	3	1
5	3	5	
6	4	7	

C4: 1 Menu

Figure 5: Handheld spreadsheet

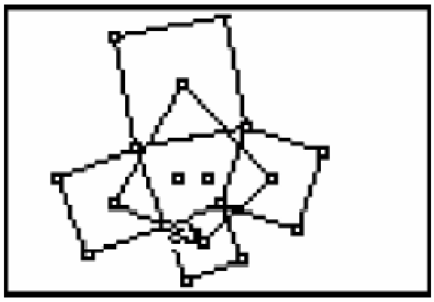


Figure 6: Handheld interactive geometry

What, then, of the possibilities regarding this merging of research domains? Two particular features of handheld graphing technology appear relevant here: their accessibility and availability to students within normal classroom situations, and the personal nature of the technology. Indeed, whole new technologies become available. The spreadsheet available for the *TI-89/92Plus/Voyage 200* series of CAS calculators appears unique in its symbolic algebra capabilities: it appears to be the first truly symbolic spreadsheet available!

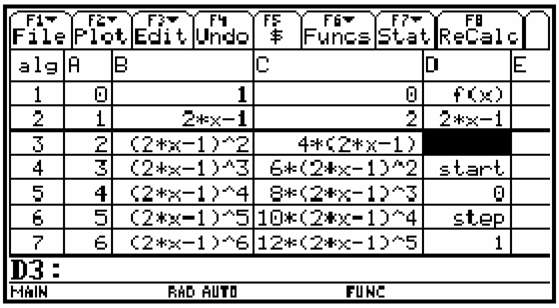


Figure 7: Handheld symbolic spreadsheet

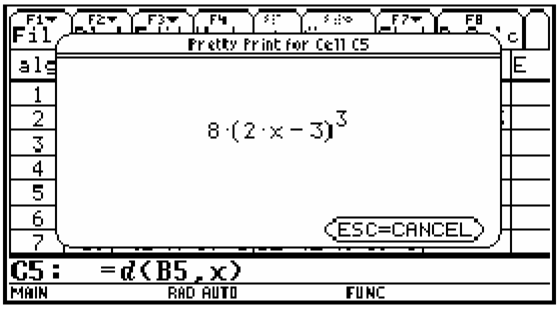


Figure 8: CAS spreadsheet

Add, then, cross-curricular capabilities, such as a *Microsoft Word*-compatible word processor (and full-sized keyboard), a variety of databases which include periodic tables, world geography and dictionaries, interactive textbooks and teaching sequences, and the line separating these devices from their more fully-featured counterparts becomes distinctly blurred. Even such simple software as the organiser, allowing students to store their friends' contact details, their "to-do list" and a calendar for their timetable, suggests that the humble "graphing calculator" may soon be fulfilling multiple roles, in school and out of school.

Figure 9: Laptop or calculator?

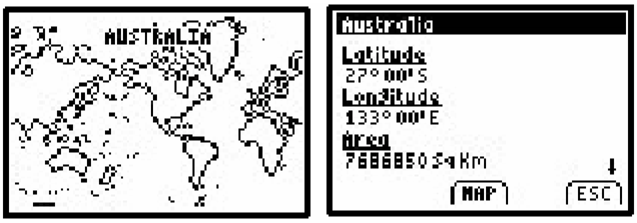


Figure 10: Cross-curricular applications

What are the research implications of a device which crosses curricular boundaries, which potentially becomes an extension of the users' self, and yet which remains fundamentally a tool for doing and learning mathematics? Imagine if these amazing tools could become communication devices as well!

Entering the Wireless World

While we are unlikely to see classroom learning devices merge with mobile phones just yet, options for wireless networking are already available. The release this year of the *Texas Instruments TI-Navigator* system offers wireless networking options for users of handheld graphing tools. Students walk into class, plug their handhelds into wireless base stations, and connect seamlessly with the teacher's computer at the front of the room. In addition to streamlining the distribution of files, this system supports instant feedback for student assessment and, perhaps most significantly, ready sharing of student work.

The potential of this last feature is being explored through the *SimCalc Project*, under the directorship of James Kaput. Using *MathWorlds* software, students create and manipulate real-time simulations which link motion to graphical forms. The software even supports physical motion from the students to be collected using a motion detector, and then modelled graphically. The possibilities for students to create and share their own motions with others, challenging them to match the motion themselves, both physically and graphically, are truly exciting.

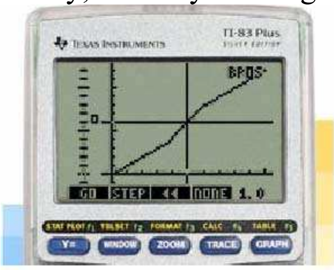


Figure 11: Modelling real motion

Of all the wonderful new technologies discussed to this point, it may well be this convergence of handheld technology, physical involvement, cognitive challenge and social empowerment that offers the best glimpse of where we might all like the mathematics classrooms of 2010 to be!

Conclusion

What, then, might our research agenda look like over the next few years of this firsr decade of a new millennium? Certainly, it must acknowledge the full power and capabilities of handheld graphing tools for mathematics teaching and learning. To this point, we have not moved beyond researching the technology as it was fifteen years ago; we must design questions and approaches which reflect the technologies of *today*.

As shown by the *SimCalc* program, it is possible to design research which looks to the future, not to the past: which explores possibilities of what *could be*, not just *what is* and *has been*. Once again, credit should go to the team at the University of Melbourne under the leadership of Professor Kaye Stacey. Following on from their *CAS-CAT Project* of the past three years, this year they begin a new research program in partnership with Victorian schools which seeks to address exactly this goal: to identify the most exciting technologies available for mathematics teaching and learning today, and carefully examine their possibilities and potential in real schools and with real classrooms.

Good research will always reflect the concerns and priorities of schools and teachers, not of researchers. It will be designed so as to gain maximum information which will inform our classroom practice and guide us in our planning and policy decisions for the future. Good research will ask the questions that matter, and seek answers which are clear, robust and, where possible, generalisable. Technology, as a field of enquiry and practise, changes far too rapidly for any of us to make firm predictions beyond the next few years; nonetheless, it is our responsibility to shine what light we can on the path ahead, so that others may walk confidently towards the classrooms of tomorrow.

References

Adams, T. L. (1997). Addressing Students' Difficulties with the Concept of Function: Applying Graphing Calculators and a Model of Conceptual Change. *Focus on Learning Problems in Mathematics, 19*(2), 43-57.

Arnold, S. (1999). CBL: Crossing the Boundaries. *Australian Mathematics Teacher, 55*(2), 4-7.

Ball, L. (2002). So, what do I write down? I've solved the whole problem using CAS. In J. Bohm (Ed.), *VISIT-ME 2002, Proceedings of the Vienna International Symposium on Integrating Technology in Mathematics Education* (Vol. SR-31). Vienna, Austria: BK-Teachware Series.

Ball, L., & Stacey, K. (2003). What should students record when solving problems with CAS? Reasons, information, the plan and some answers. In J. T. Fey, A. Cuoco, C. Kieran, L. Mullin & R. M. Zbiek (Eds.), *Computer algebra systems in secondary school mathematics education* (pp. 289-303). Reston, Virginia: National Council of Teachers of Mathematics.

Ball, L., Stacey, K., & Pierce, R. (2001). *Assessing algebraic expectation.* Paper presented at the Proceedings of the Twenty-Fourth Annual Conference of the Mathematics Education Research Group of Australasia, Sydney.

Berger, M. (1998). Graphic Calculators: An Interpretive Framework. *For the Learning of Mathematics, 18*(2), 13-20.
Boers, M. A., & Jones, P. L. (1994). Students' use of graphic calculators under examination conditions. *International Journal of Mathematical Education in Science & Technology, 25*, 491-516.
Bosche, W. W., Jr. (1997). *Impact of a nontraditional technology-enhanced approach on student conceptualization in a precalculus course.* Unpublished PhD, Georgia State University.
Brueningsen, C., & Bower, W. (1995). Using the Graphing Calculator--in Two-Dimensional Motion Plots. *Physics Teacher, 33*(5), 314-316.
Burrill, G. (1996). *Graphing calculators and their potential for teaching and learning statistics.* Paper presented at the Research on the Role of Technology in Teaching and Learning Statistics, Grenada, Spain.
Burrill, G., Allison, J., Breaux, G., Kastberg, S., Leatham, K., & Sanchez, W. (2003). *Handheld graphing technology in secondary mathematics: Research findings and implications for classroom practice*: Michigan State University.
Caldwell, F. (1997). *Bring Functions and Graphs to Life with the CBL.* Paper presented at the the Carolinas Mathematics Conference (Charlotte, NC, October 23, 1997).
Carson, V. M. (1995). *Data Analysis to Explore the Sampling Distribution of the Sample Mean (Mathematics).* Unpublished PHD, Georgia State University.
Cates, B. B. (2000). *The effects of Calculator-Based Laboratory activities on college algebra students' understanding of the function concept and graphing.* Unpublished PhD, North Carolina State University.
Connors, M. A., & Snook, K. G. (2001). The Effects of Hand-Held CAS on Student Achievement in a First Year College Core Calculus Sequence. *International Journal of Computer Algebra in Mathematics Education, 8*(2), 99-114.
Creed, F. H., III. (2000). *A comparative study of the use of spreadsheet software and graphing calculator technology in introductory statistics.* Unpublished PhD, Georgia State University.
Dahland, G., & Lingefjard, T. (1996). Graphing calculators and students' interpretation of results. *Nordic Studies in Mathematics Education, 4*(2-3), 31-50.
Doerr, H. M., & Zangor, R. (1999). The Teacher, the Task and the Tool: The Emergence of Classroom Norms. *International Journal of Computer Algebra in Mathematics Education, 6*(4), 267-279.
Doerr, H. M., & Zangor, R. (2000). Creating Meaning for and with the Graphing Calculator. *Educational Studies in Mathematics, 41*(2), 143-163.
Drijvers, P. (2000). Students encountering obstacles using a CAS. *International Journal of Computers for Mathematical Learning, 5*(3), 189-209.
Drijvers, P., & Doorman, M. (1996). The Graphics Calculator in Mathematics Education. *Journal of Mathematical Behavior, 15*(4), 425-440.
Drijvers, P., & van Herwaarden, O. (2000). Instrumentation of ICT-tools: The case of algebra in a computer algebra environment. *International Journal of Computer Algebra in Mathematics Education, 7*(4), 255-276.
Emerson, D. K. (2000). *An evaluation of the Fathom software as a tool for the teaching of statistics with specific comparisons to the use of a graphing calculator.* Unpublished MS, Texas Woman's University.
Farrell, A. M. (1996). Roles and Behaviors in Technology-Integrated Precalculus Classrooms. *Journal of Mathematical Behavior, 15*(1), 35-53.
Fleener, M. J. (1995a). The relationship between experience and philosophical orientation: A comparison of preservice and practicing teachers' beliefs about calculators. *Journal of Computers in Mathematics & Science Teaching, 14*(3), 359-376.
Fleener, M. J. (1995b). A survey of mathematics teachers' attitudes about calculators: The impact of philosophical orientation. *Journal of Computers in Mathematics & Science Teaching, 14*(4), 481-498.
Forster, P. (2000). Process and Object Interpretations of Vector Magnitude Mediated by Use of the Graphics Calculator. *Mathematics Education Research Journal, 12*(3), 269-285.
Forster, P., Flynn, P., Frid, S., & Sparrow, L. (2004). Calculators and computer algebra systems. In *MERGA review of research in mathematics education*. Melbourne: MERGA.
Forster, P., & Mueller, U. (2001). Outcomes and Implications of Students' Use of Graphics Calculators in the Public Examination of Calculus. *International Journal of Mathematical Education in Science & Technology, 32*(1), 37-52.
Forster, P., & Taylor, P. (2000). A Multiple-Perspective Analysis of Learning in the Presence of Technology. *Educational Studies in Mathematics, 42*(1), 35-59.
Gardiner, T. (2001). Education or CAStration? *Micromath, 17*(1), 6-8.

Goos, M., & Cretchley, P. (2004). Computers, multimedia and the Internet in Mathematics Education. In *MERGA review of research in mathematics education*. Melbourne: MERGA.

Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2000). Reshaping Teacher and Student Roles in Technology-Enriched Classrooms. *Mathematics Education Research Journal, 12*(3), 303-320.

Graham, A. (1998). Waking Up to Maths with "Snap, Prattle and Pop." *Teaching Mathematics & its Applications, 17*(1), 22-27.

Graham, A., & Thomas, M. O. J. (1998). A graphic calculator approach to algebra. *Mathematics Teaching, 167*, 27-34.

Graham, A., & Thomas, M. O. J. (2000). Building a Versatile Understanding of Algebraic Variables with a Graphic Calculator. *Educational Studies in Mathematics, 41*(3), 265-282.

Green, J. (1998). Exploring Mathematical Functions with the TI-92's CBL. *Australian Senior Mathematics Journal, 12*(2), 58-63.

Guin, D., & Trouche, L. (1998). The Complex Process of Converting Tools into Mathematical Instruments: The Case of Calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195-227.

Harskamp, E. G., Suhre, C. J. M., & van Streun, A. (1998). The Graphics Calculator in Mathematics Education: An Experiment in the Netherlands. *Hiroshima Journal of Mathematics Education, 6*, 13-31.

Harskamp, E. G., Suhre, C. J. M., & van Streun, A. (2000). The Graphics Calculator and Students' Solution Strategies. *Mathematics Education Research Journal, 12*(1), 37-52.

Haruta, M., Turpin, M., & McGivney, R. (1998). Towards a New Precalculus. *Amatyc Review, 19*(2), 26-34.

Hennessy, S., Fung, P., & Scanlon, E. (2001). The Role of the Graphic Calculator in Mediating Graphing Activity. *International Journal of Mathematical Education in Science & Technology, 32*(2), 267-290.

Hollar, J. C. (1996). *The effects of a graphing approach college algebra curriculum on students' understanding of the function concept.* Unpublished PhD, North Carolina State University.

Hollar, J. C., & Norwood, K. (1999). The Effects of a Graphing-Approach Intermediate Algebra Curriculum on Students' Understanding of Function. *Journal for Research in Mathematics Education, 30*(2), 220-226.

Hong, Y. Y., Thomas, M., & Kiernan, C. (2000). Supercalculators and University Entrance Calculus Examinations. *Mathematics Education Research Journal, 12*(3), 321-336.

Huntley, M. A., Rasmussen, C. L., Villarubi, R. S., Sangtong, J., & Fey, J. T. (2000). Effects of standards-based mathematics education: A study of the Core-Plus Mathematics Project algebra and functions strand. *Journal for Research in Mathematics Education, 31*(3), 328-361.

Keller, B. (1998). Shedding Light on the Subject. *Mathematics Teacher, 91*(9), 756-762,767-771.

Keller, B., & Hirsch, C. R. (1998). Student Preferences for Representations of Functions. *International Journal of Mathematical Education in Science & Technology, 29*(1), 1-17.

Keller, B., Russell, C., & Thompson, H. (1999). A Large-Scale Study Clarifying the Roles of the TI-92 and Instructional Format on Student Success in Calculus. *International Journal of Computer Algebra in Mathematics Education, 6*(3), 191-207.

Kendal, M., & Stacey, K. (1999). Varieties of teacher privileging for teaching calculus with computer algebra systems. *International Journal of Computer Algebra in Mathematics Education, 6*, 233-247.

Lauten, A. D. (1994). Student Understanding of Basic Calculus Concepts: Interaction with the Graphics Calculator. *Journal of Mathematical Behavior, 13*(2), 225-237.

Leigh-Lancaster, D. (2000). *Curriculum and Assessment Congruence--Computer Algebra Systems (CAS) in Victoria.* Paper presented at the the T3 World-Wide Conference (Tokyo, Japan, August 6-8, 2000).

Lloyd, G. M., & Wilson, M. S. (1998). Supporting innovation: The impact of a teacher's conceptions of functions and his implementation of a reform curriculum. *Journal for Research in Mathematics Education, 29*(3), 248-274.

Merriweather, M., & Tharp, M. L. (1999). The Effect of Instruction with Graphing Calculators on How General Mathematics Students Naturalistically Solve Algebraic Problems. *Journal of Computers in Mathematics & Science Teaching, 18*(1), 7-22.

Mitchelmore, M., & Cavanagh, M. (2000). Students' Difficulties in Operating a Graphics Calculator. *Mathematics Education Research Journal, 12*(3), 254-268.

Nicol, M. L. P. (1995). *Examining the Changing Beliefs of a High School Physics Teacher Integrating Mathematics through Technology: A Case Study.* Unpublished PHD, The Ohio State University.

Penglase, M., & Arnold, S. (1996). The Graphics Calculator in Mathematics Education: A Critical Review of Recent Research. *Mathematics Education Research Journal, 8*(1), 58-90.

Pierce, R., & Stacey, K. (2001). Reflections on the changing pedagogical use of computer algebra systems: Assistance for doing or learning mathematics? *Journal for Computing in Mathematics and Science Teaching, 20*(2), 143-162.

Porzio, D. T. (1999). Effects of differing emphases in the use of multiple representations and technology on students' understanding of calculus concepts. *Focus on Learning Problems in Mathematics, 21*(3), 1-29.

Quesada, A. R., & Maxwell, M. E. (1994). The Effects of Using Graphing Calculators to Enhance College Students' Performance in Precalculus. *Educational Studies in Mathematics, 27*(2), 205-215.

Rochowicz Jr., J. A. (1996). The impact of using computers and calculators on calculus instruction. *Journal of Computers in Mathematics & Science Teaching, 15*, 423-435.

Ruthven, K. (1990). The Influence of Graphic Calculator Use on Translation from Graphic to Symbolic Forms. *Educational Studies in Mathematics, 21*(5), 431-450.

Schwarz, B. B., & Hershkowitz, R. (1999). Prototypes: Brakes or levers in learning the function concept? The role of computer tools. *Journal for Research in Mathematics Education, 30*(4), 362-389.

Shoaf-Grubbs, M. M. (1992). *The Effect of the Graphics Calculator on Female Students' Cognitive Levels and Visual Thinking.* Unpublished PHD, Columbia University.

Slavit, D. (1996). Graphing Calculators in a "Hybrid" Algebra II Classroom. *For the Learning of Mathematics, 16*(1), 9-14.

Slavit, D. (1998). Three women's understanding of algebra in a precalculus course integrated with the graphing calculator. *Journal of Mathematical Behavior, 17*(3), 303-389.

Stacey, K., Kendal, M., & Pierce, R. (2002). Teaching with CAS in a Time of Transition. *International Journal of Computer Algebra in Mathematics Education, 9*(2), 113-127.

Tharp, M. L., Fitzsimmons, J. A., & Ayers, R. L. B. (1997). Negotiating a Technological Shift: Teacher Perception of the Implementation of Graphing Calculators. *Journal of Computers in Mathematics & Science Teaching, 16*(4), 551-575.

Thompson, D. R., & Senk, S. L. (2001). The effects of curriculum on achievement in Second-Year Algebra: The examples of the University of Chicago School Mathematics Project. *Journal for Research in Mathematics Education, 32*, 58-84.

van Streun, A., Harskamp, E., & Suhre, C. (2000). The Effect of the Graphic Calculator on Students' Solution Approaches: A Secondary Analysis. *Hiroshima Journal of Mathematics Education, 8*, 27-39.

Weiss, I., Banilower, E., & Smith, P. S. (2001). *Report of the 2000 national survey of science and mathematics education.* Chapel Hill NC: Horizon Research, Inc.

White, J. W., & Norwich, V. H. (1997). *Computer Activities for College Algebra and Precalculus* (No. ED412119).

Zbiek, R. M. (1998). Prospective teachers' use of computing tools to develop and validate functions as mathematical models. *Journal for Research in Mathematics Education, 29*(2), 184-201.

Zimmermann, G. M. (2002). *Students' reasoning about probability simulations during instruction.* Unpublished PhD, Illinois State University.